Probabilistic Mapping

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based on presentation by
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Motivation

- Map creation – more measurements required
- Measurements are never accurate
  - System measurements:
    - Surfaces change
    - Sensor design (cone instead of line)
  - Environment measurements
    - Wind, humidity and temperature
    - Material properties

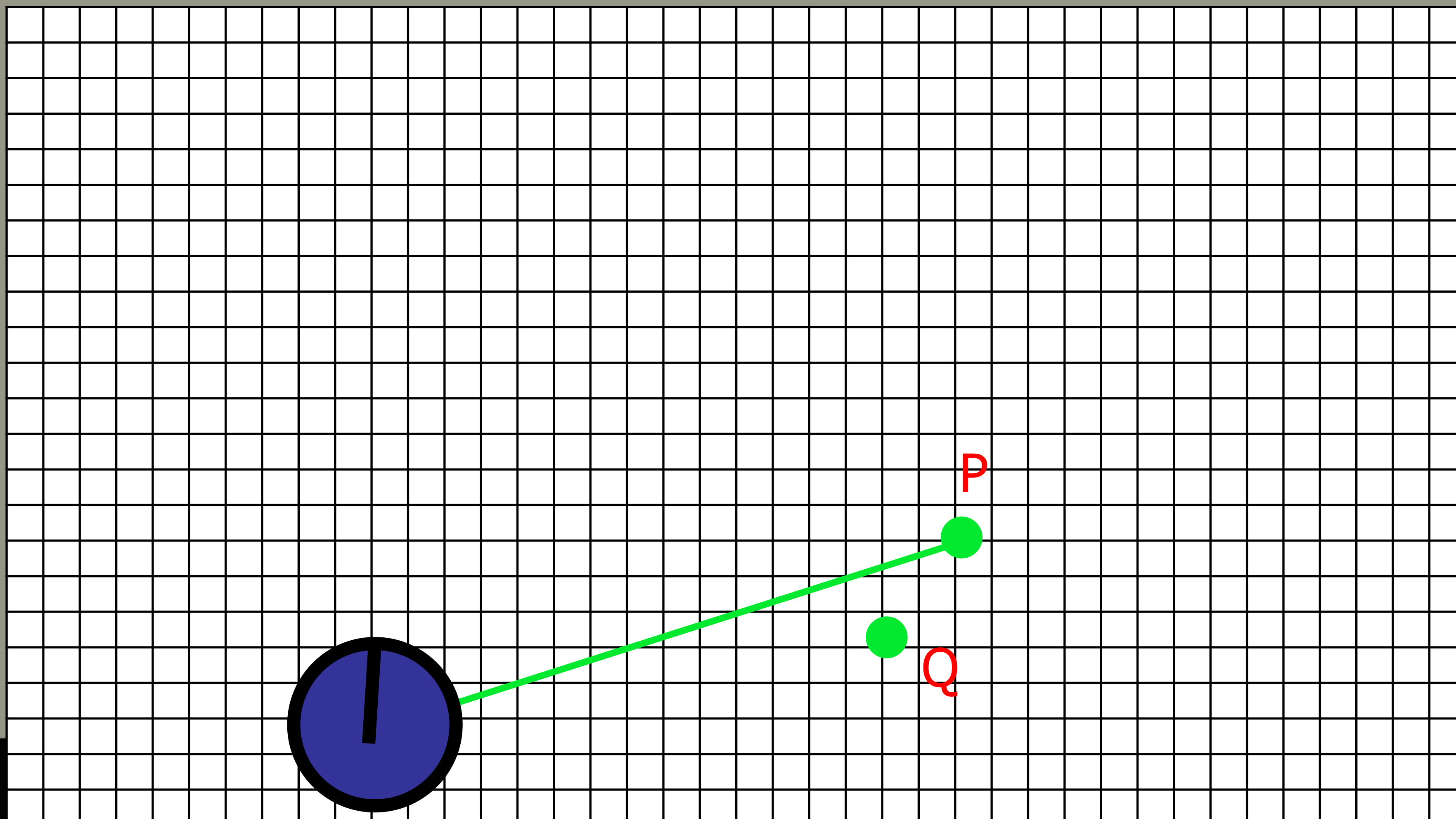
Riešenie

- Vytvoríme **pravdepodobnostné** modely:
  - Senzoru
  - Prostredia

Sonar Model

- Pravdepodobnostný model nám umožní zahrnúť aj neurčitosť merania
- Povedzme, získame údaj 2,23 m v smere 70°:
  - Ak sme toto namerali, aká je pravdepodobnosť, že v mieste [21,2 42,1] je nejaká prekážka?
- Ak máme pri znalosti neurčitosti senzora nejaké meranie v bode P, čo vieme povedať o Q?

Sonar Model



Sonar Model

- Aké vlastnosti očakávame od vytvoreného modelu senzora?

Sonar Model

8	3						4	
4			7			8	5	
	5							
9			3				8	2
		8			1			
				6	5		3	
3	4		2		7			
	6	7			9			

Sonar Model

8	3						4	
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Sonar Model

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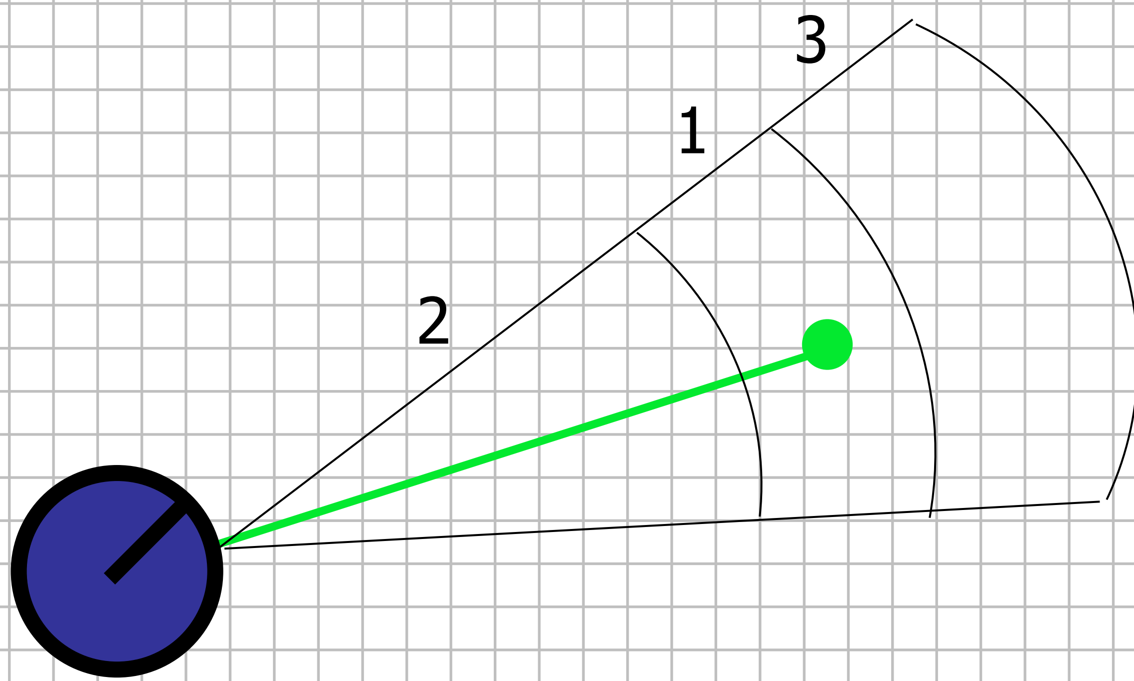
Sonar Model

8	3						4	
4	-		7			8	5	
	5							
	-							
9	-		3				8	2
	-	8			1			
x	8	x	x	6	5	x	3	x
3	4	x	2		7			
x	6	7			9			

Sonar Model

- Simplest model



Sonar Model

- Aké vlastnosti očakávame od vytvoreného modelu senzora?

- Budeme pracovať s mriežkou obsadenosti, tzv. *occupancy grid*.
- Prekážka = 1, prázdne miesto = 0
- Čo znamená hodnota 0,5?

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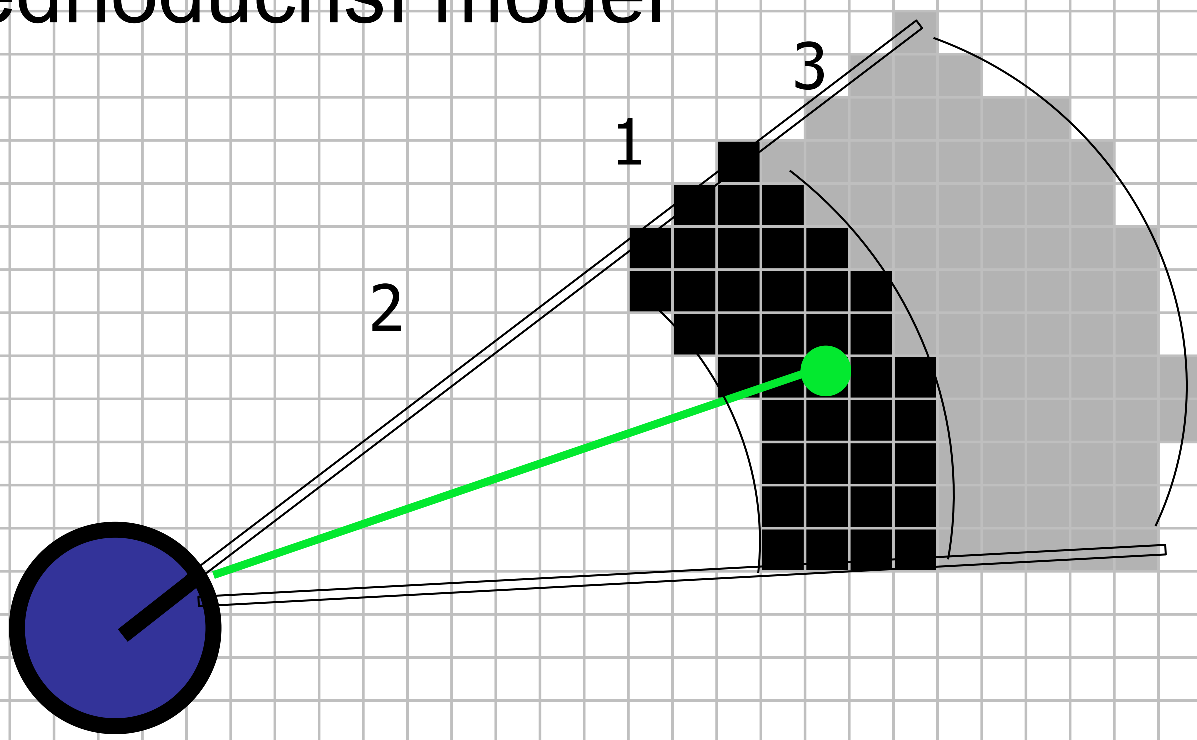
Sonar Model

Najjednoduchší model



Sonar Model

- Aké vlastnosti očakávame od vytvoreného modelu senzora?

- Budeme pracovať s mriežkou obsadenosti, tzv. *occupancy grid*.
- Prekážka = 1, prázdne miesto = 0
- Čo znamená hodnota 0,5?

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Sonar Model

- Oblasť medzi objektom a senzorom: Region 2
- je voľná, pravdepodobnosť obsadenia políčok v tejto oblasti je **0**

- Oblasť v okolí zmeranej vzdialenosti: Region 1
  - neistota zmeranej vzdialenosti
  - neistota v uhle
- pravdepodobnosť obsadenia políčok v tejto oblasti je **1**

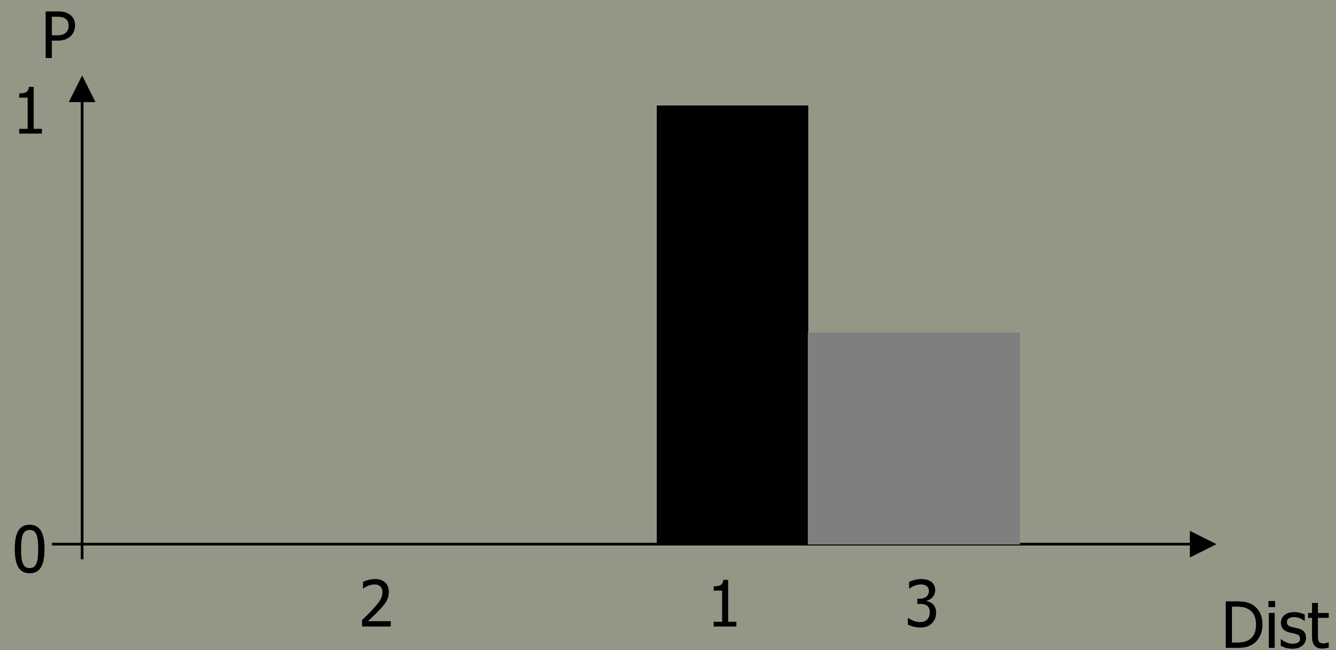
Sonar Model

- O oblasti, ktorá je skrytá ZA prekážkou, nemáme žiadne informácie: Region 3

- pravdepodobnosť obsadenia políčok v tejto oblasti je **0,5**
- ak máme informáciu vopred, môžeme ju zmeniť, napr. pokrytie povrchu Marsu skalami je 75%, takže p = **0,75**

Sonar Model

- Triviálny model



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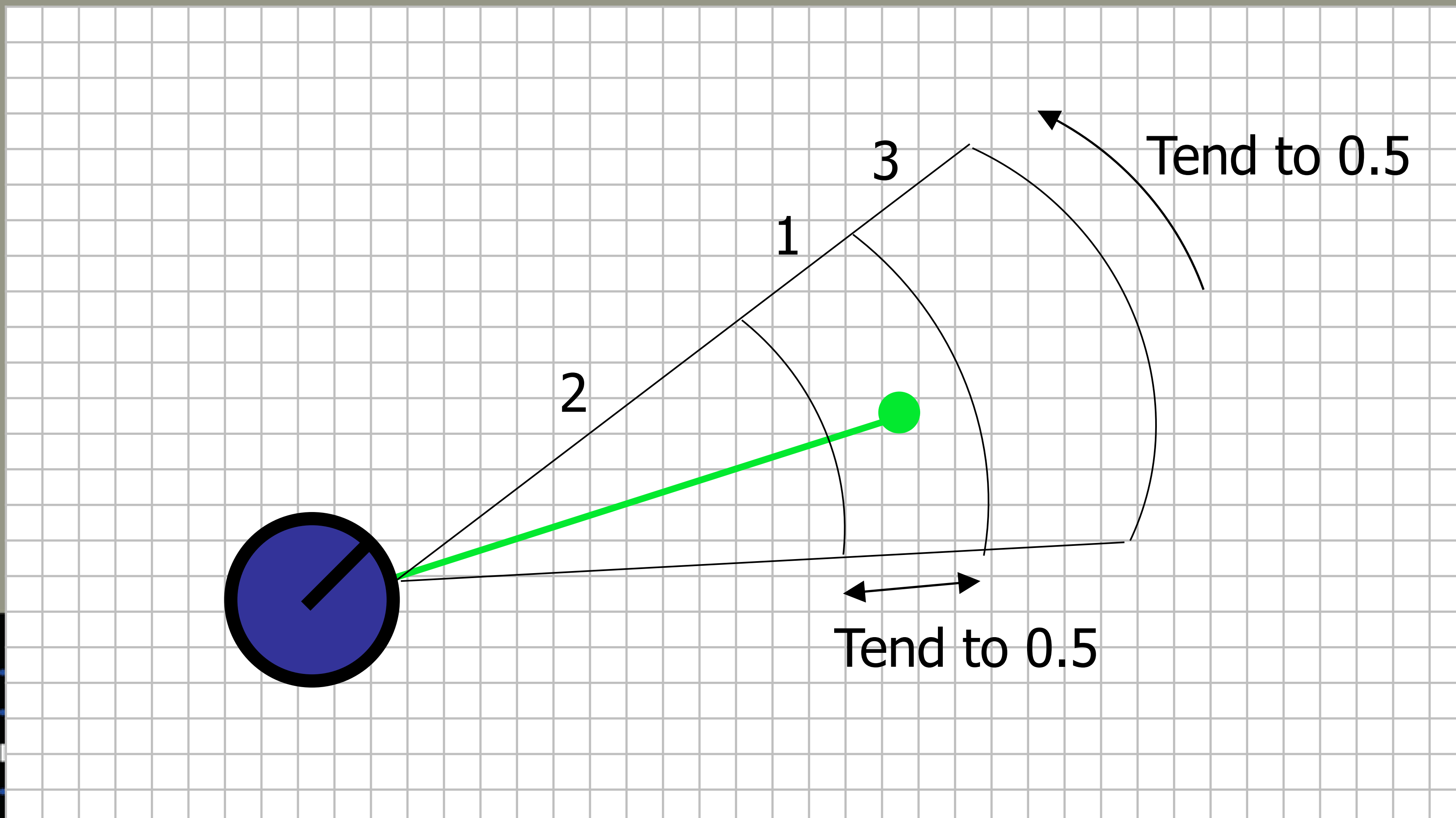
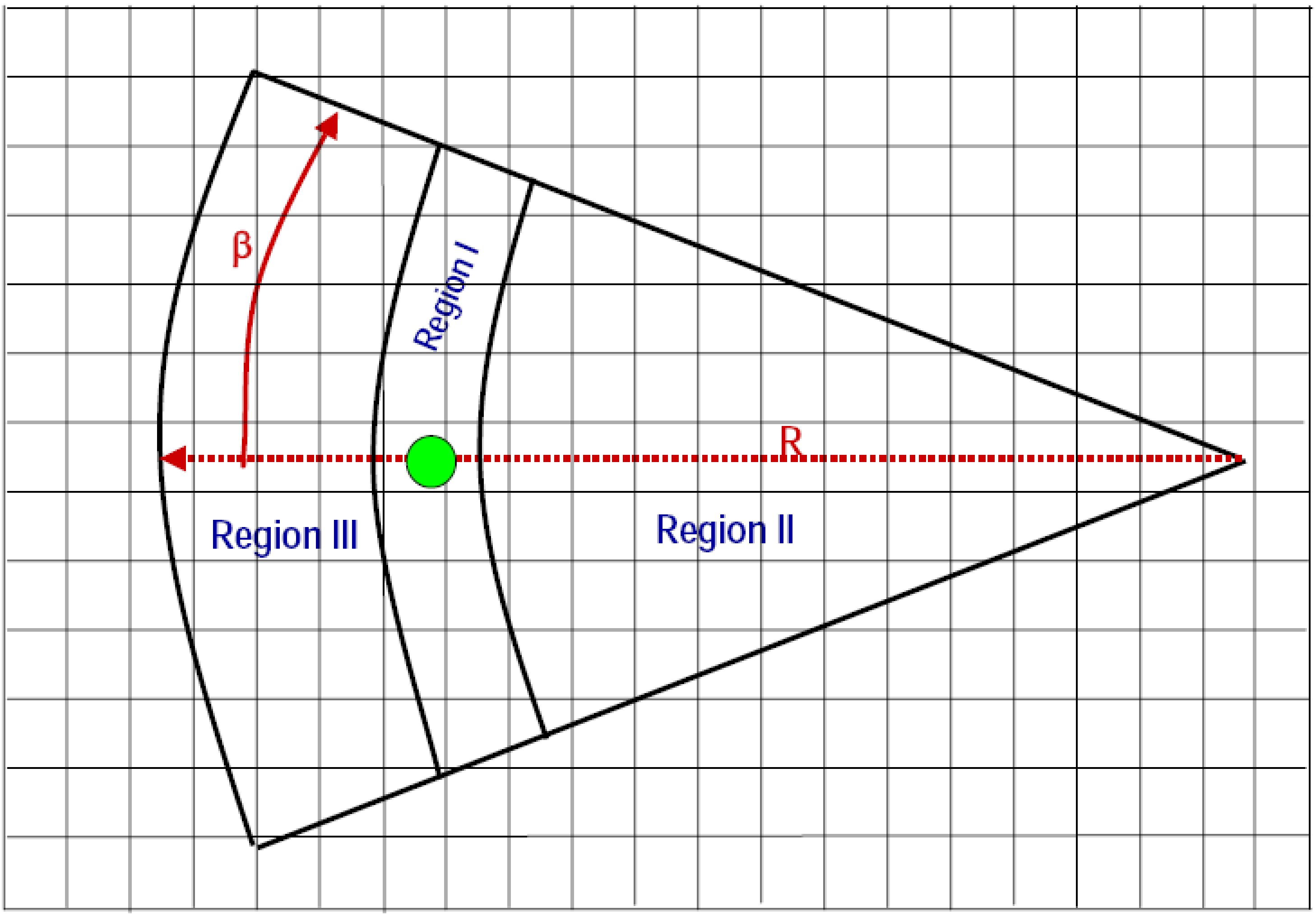
Sonar Model

- Vylepšujeme model



Modeling Common Sonar Sensor

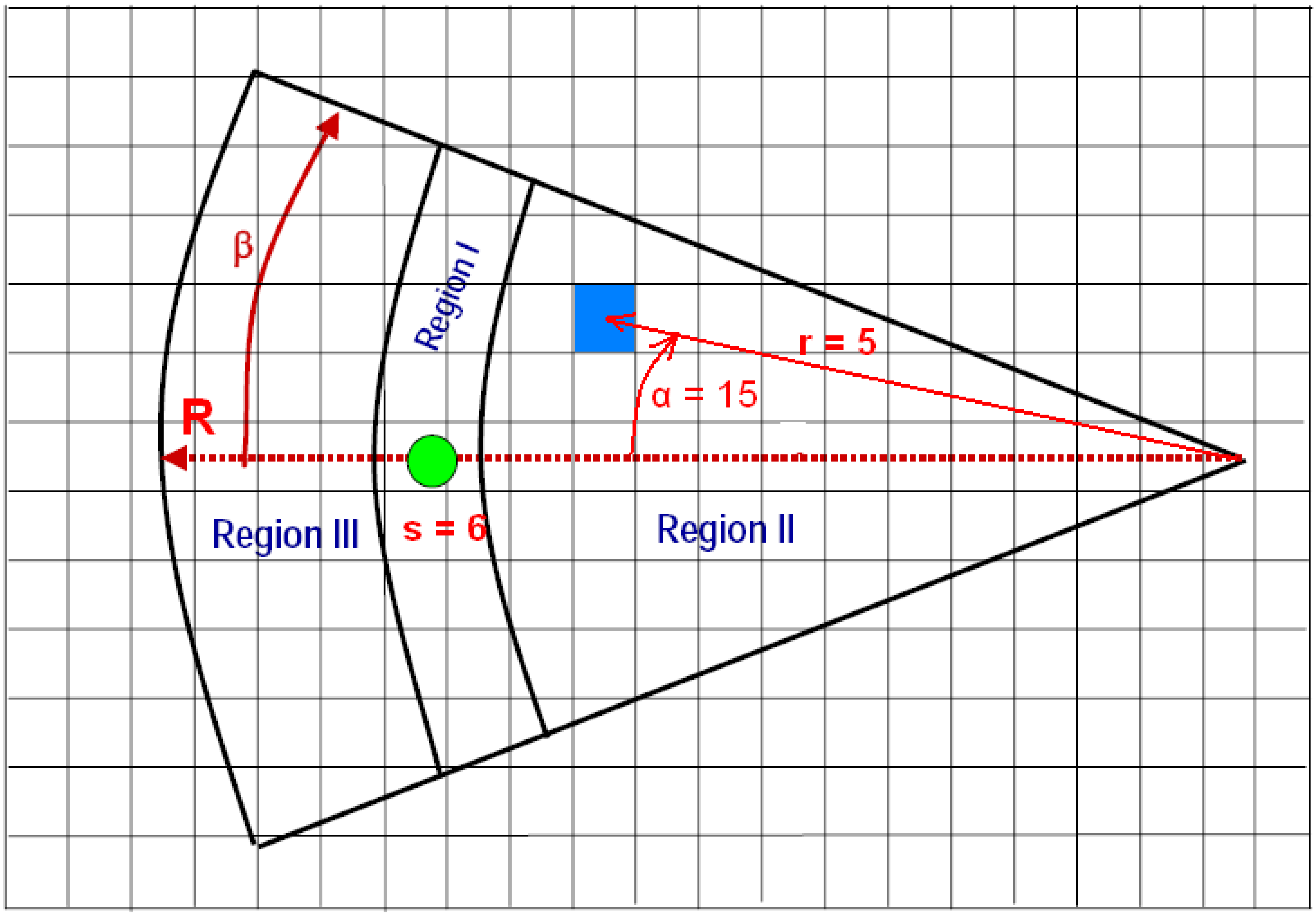


Region I: Probably occupied

Region II: Probably empty

Region III: Unknown

Modeling Common Sonar Sensor



Region I: Probably occupied   Region II: Probably empty   Region III: Unknown

S T F

Converting Sonar Reading to Probability: Region I

- Region I:

The nearer the grid element to the origin of the sonar beam, the higher the belief

The closer to the acoustic axis, the higher the belief

$$P(Occupied) = \frac{\frac{R-r}{R} + \frac{\beta-\alpha}{\beta}}{2} \text{ x } Max_{occupied}$$

We never know with certainty

where r is distance to grid element,

α is angle to grid element

$Max_{occupied}$ = highest probability possible (e.g., 0.98)

$$P(Empty) = 1.0 - P(Occupied)$$

Converting Sonar Reading to Probability: Region II

- Region II:

The nearer the grid element to the origin of the sonar beam, the higher the belief

The closer to the acoustic axis, the higher the belief

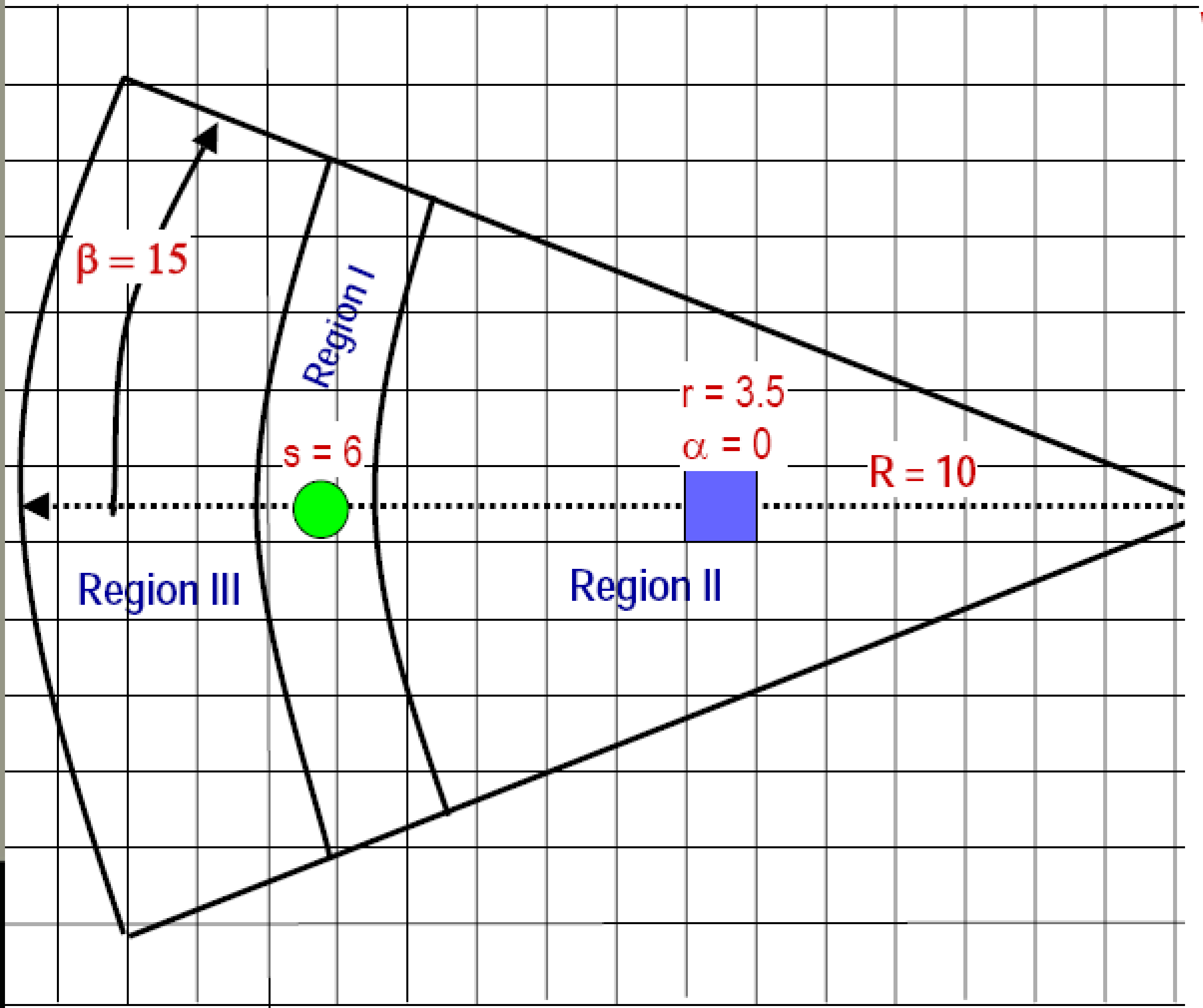
$$P(Empty) = \frac{\frac{R-r}{R} + \frac{\beta-\alpha}{\beta}}{2}$$

$$P(Occupied) = 1.0 - P(Empty)$$

where r is distance to grid element,

α is angle to grid element

Example: What is value of grid cell ?



Which region?

3.5 < (6.0 – 0.5) ➔ Region II

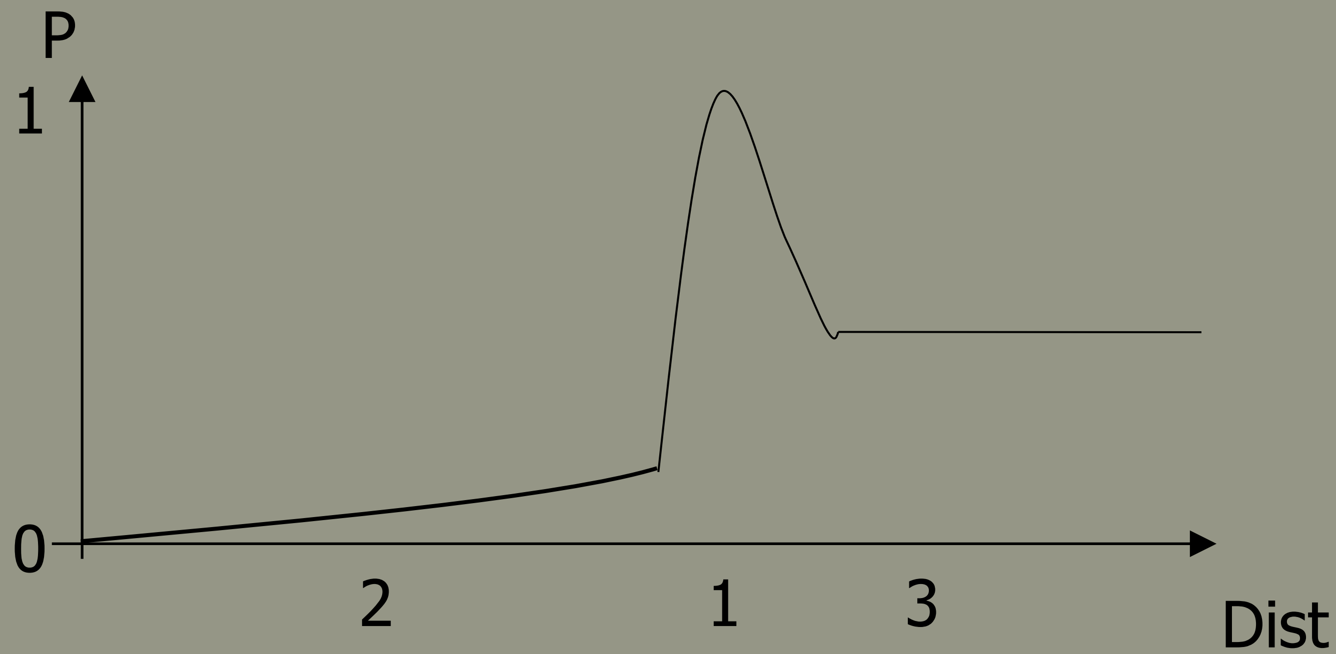
$$P(\text{Empty}) = \frac{\frac{10-3.5}{10} + \frac{15-0}{15}}{2}$$

$$= 0.83$$

P(Occupied) = (1 – 0.83) = 0.17

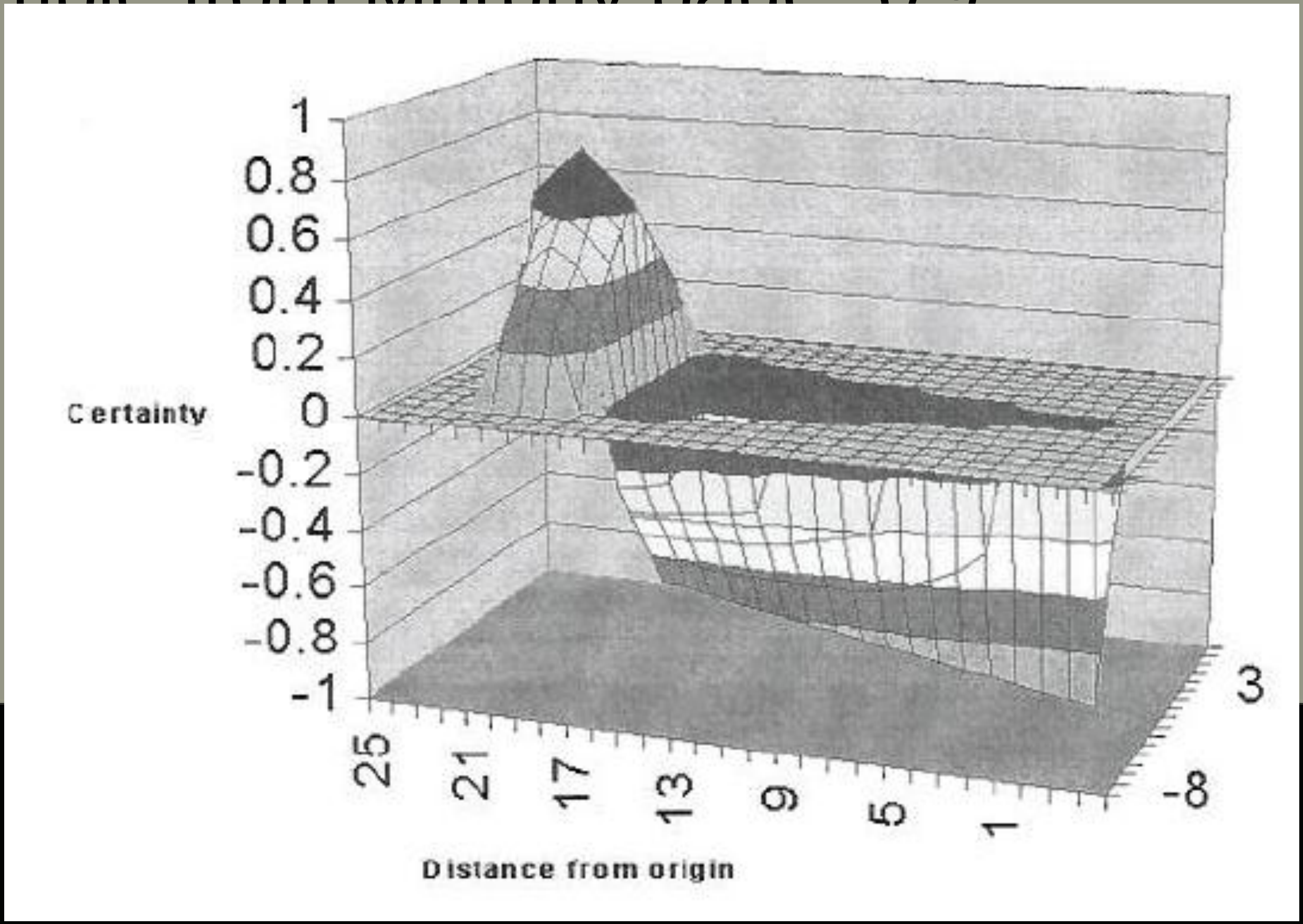
Sonar Model

- More complex model



Sonar Model

- Example from Murphy page 379

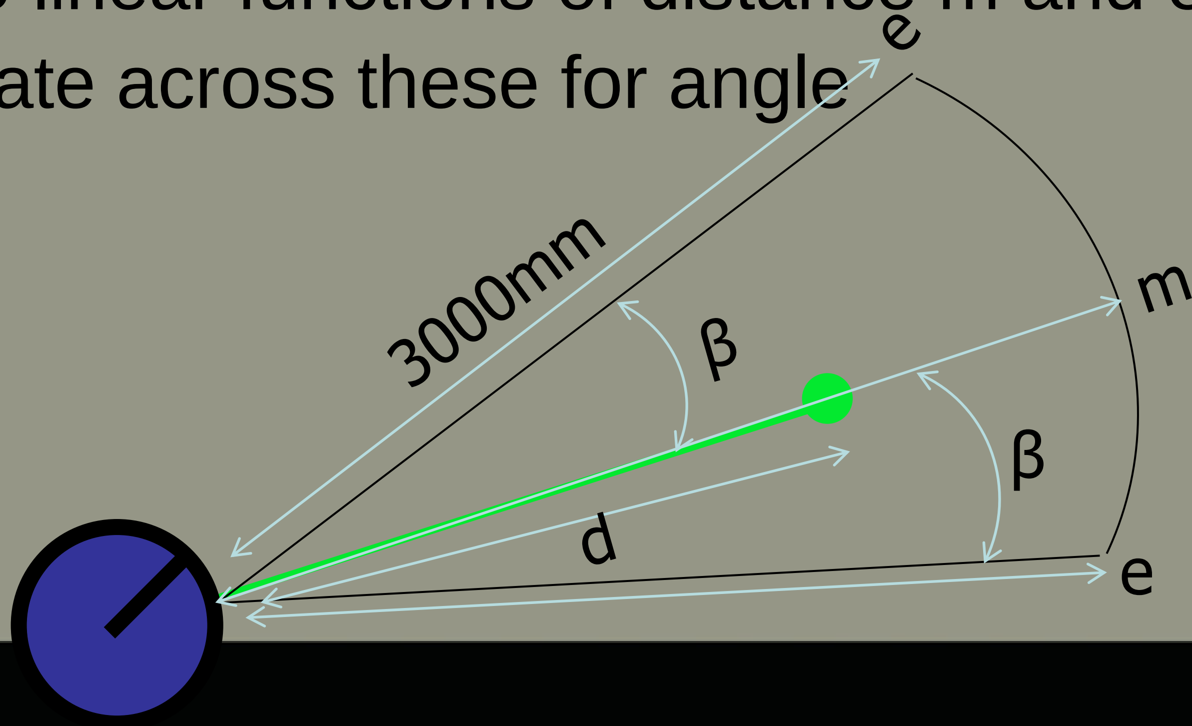


Sonar Model

- How to get create a model like this?
  - Use parametric functions
    - Bell curve
    - Triangle
    - Trapezoid
  - Parameters
    - Angle
    - Distance
  - Combine these functions and plot in polar space

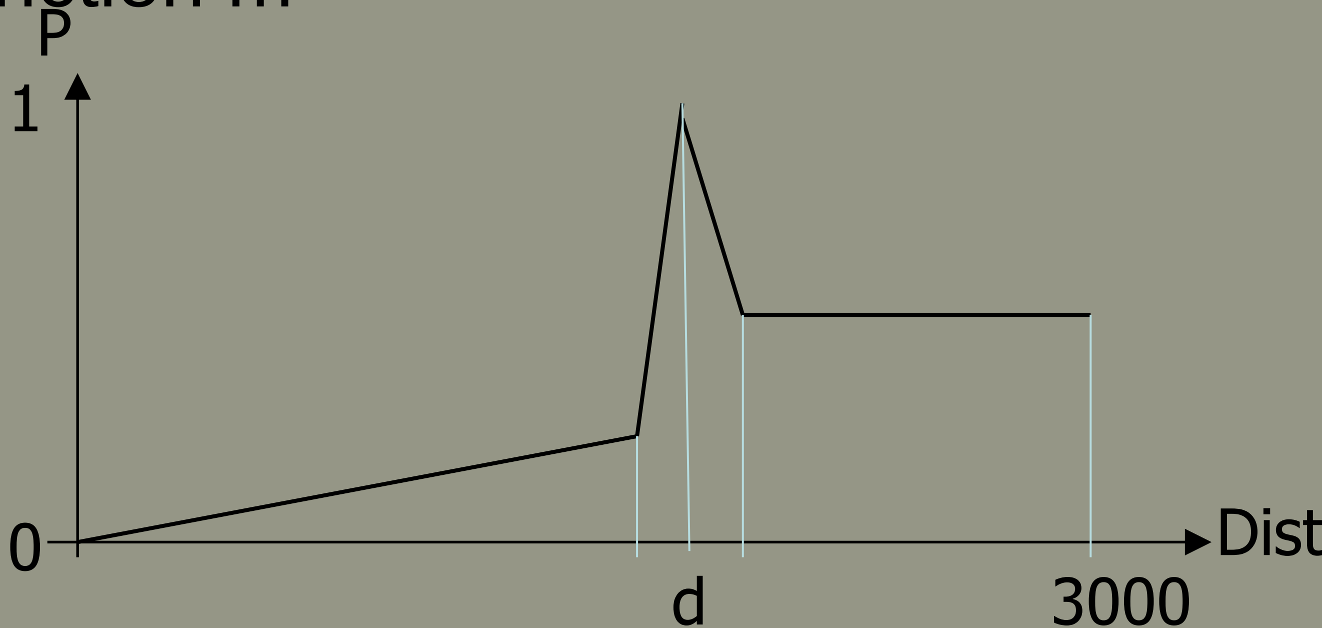
Sonar Model

- Example using linear interpolation
  - Use two linear functions of distance m and e
  - Interpolate across these for angle



Sonar Model

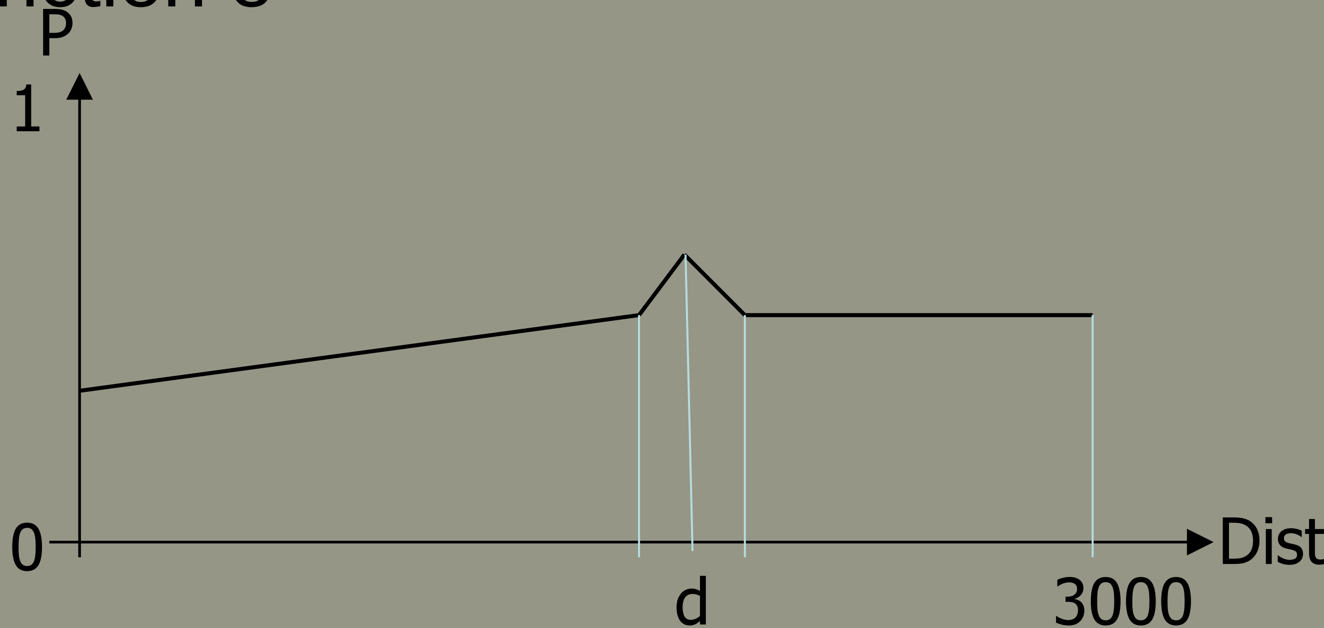
- Function m



- Points = {(0,0),(max(0,d-150),0.25), (d,1.0), (min(d+150,3000),0.5), (3000,0.5)}

Sonar Model

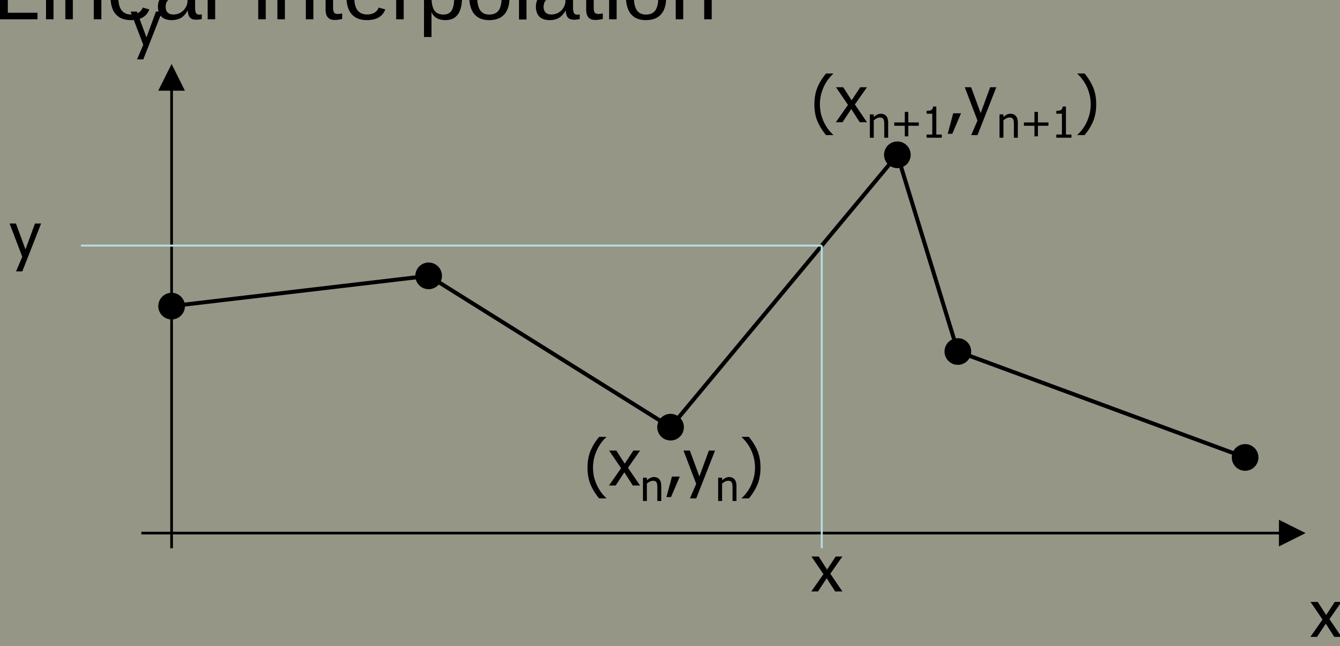
- Function e



- Points = {(0,0.4),(max(0,d-150),0.5), (d,0.6), (min(d+150,3000),0.5), (3000,0.5)}

Sonar Model

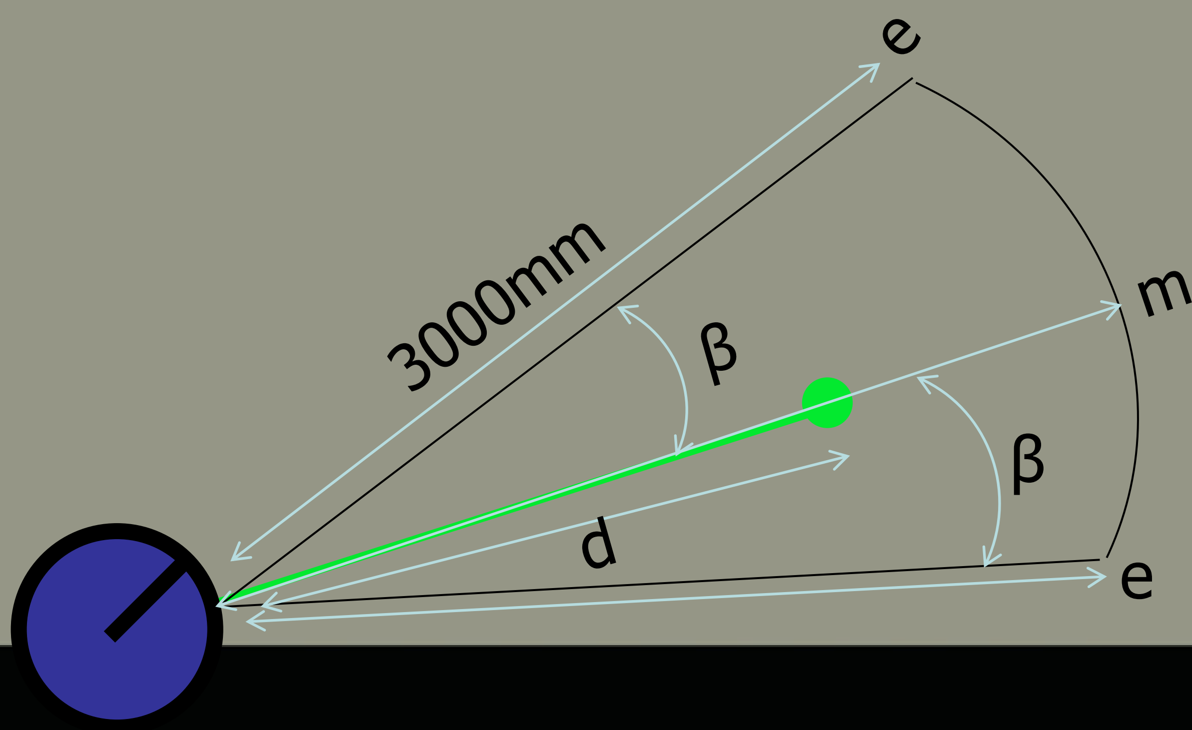
- Linear interpolation



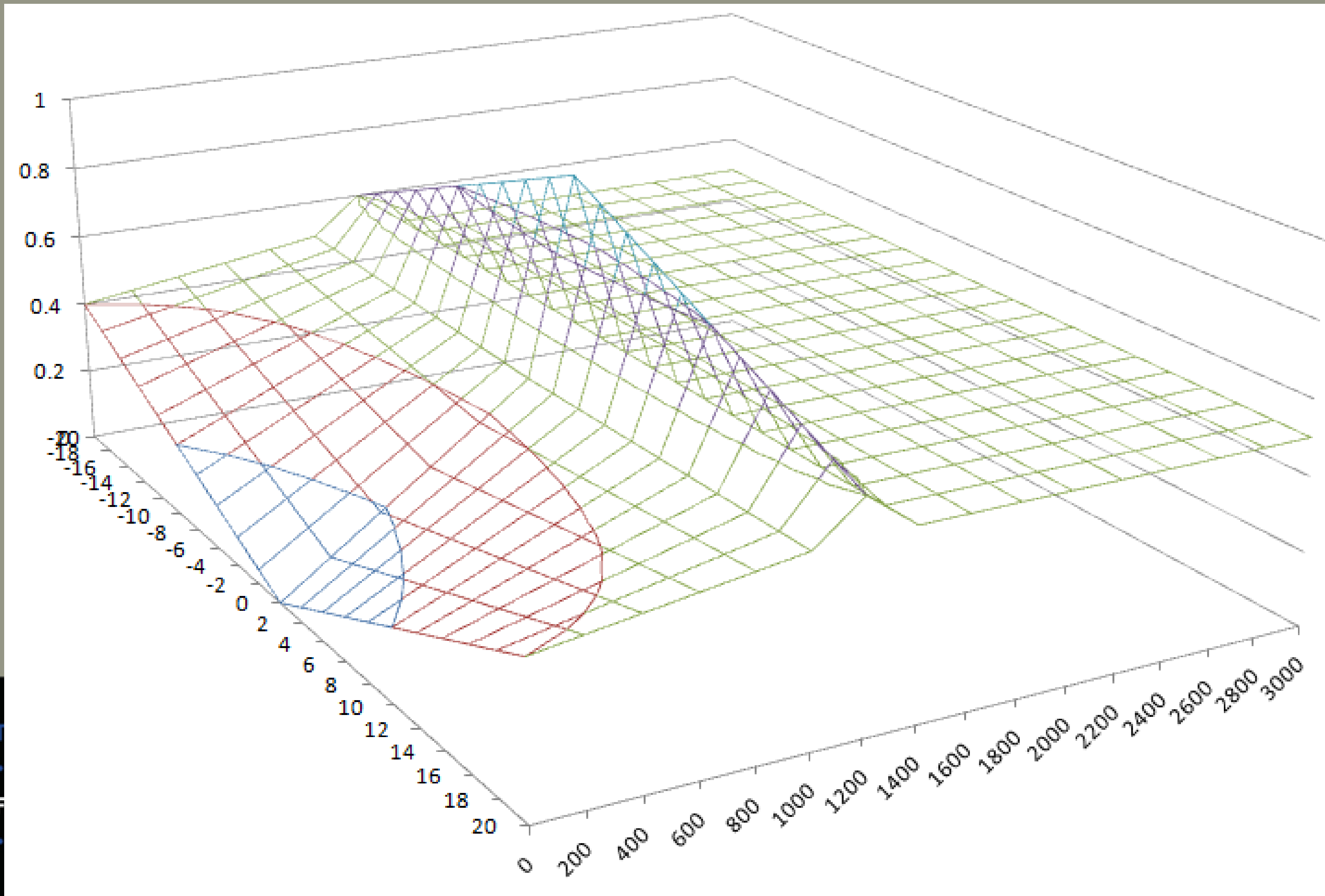
$$y = y_n + (x - x_n) \times \frac{y_{n+1} - y_n}{x_{n+1} - x_n}$$

Sonar Model

- Recall

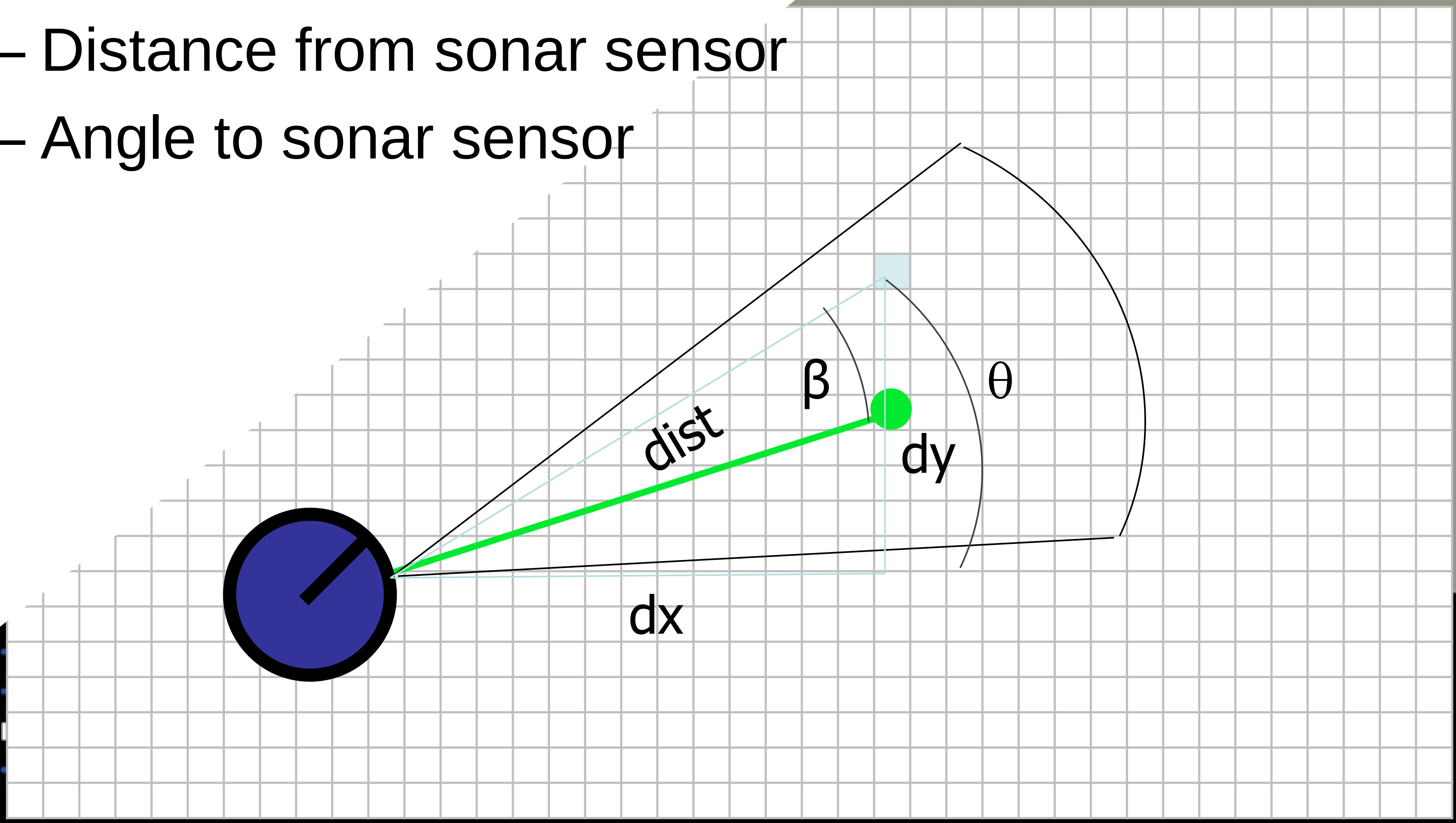


In Cartesian Co-ord Space



Polar Space

- For each grid square we calculate:
  - Distance from sonar sensor
  - Angle to sonar sensor



Polar Space

- Solutions:

$$dist = \sqrt{dx^2 + dy^2}$$

$$\theta = arctan2(dy, dx)$$

$$\beta = sonar\theta - \theta$$

Sonar Model

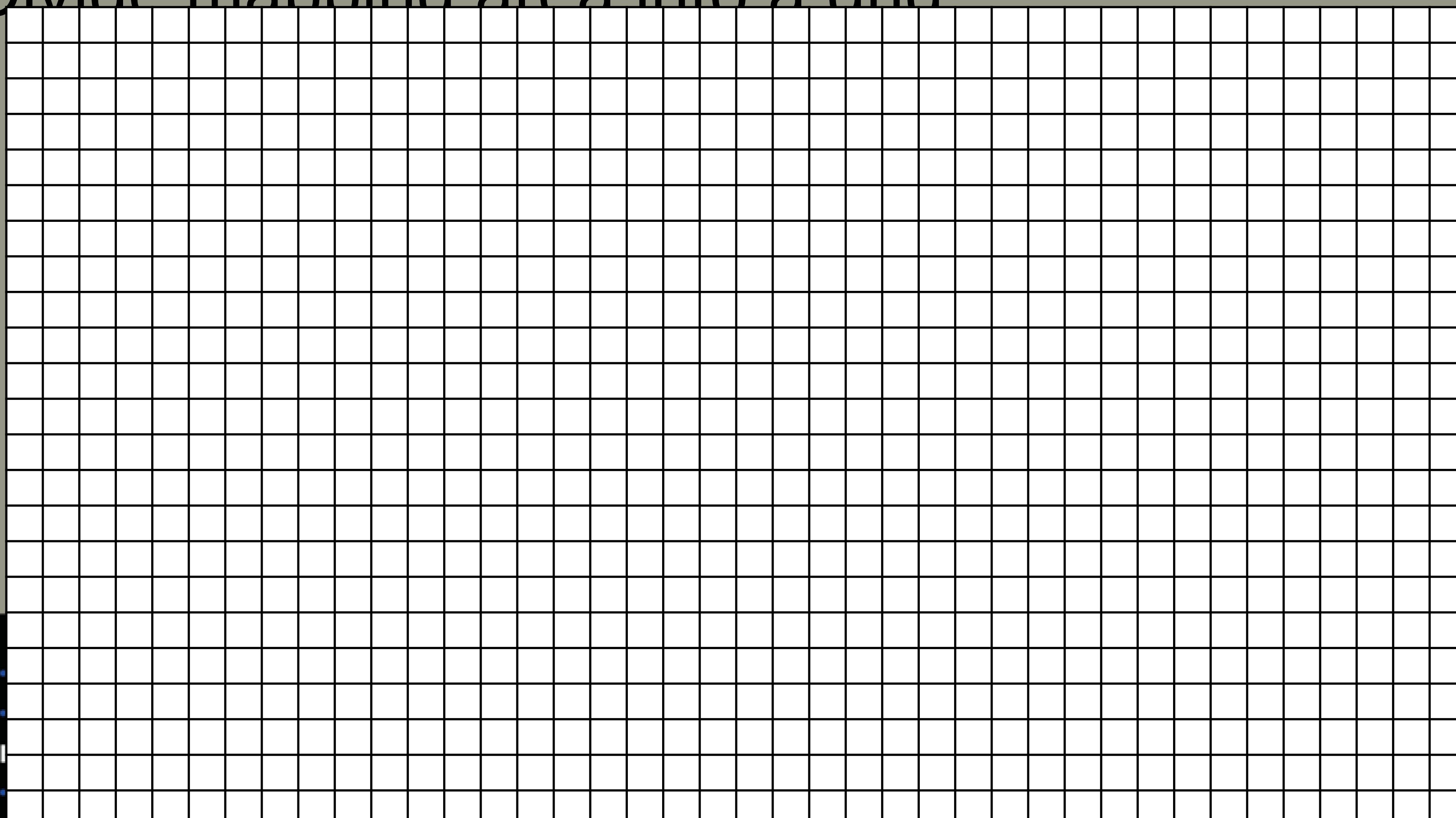
- Demo

The Occupancy Grid

- Current map model is binary:
  - A point in space is either occupied or not
  - True whether scattergram or a line based map
- Range readings are binary
- However there are uncertainties:
  - Wind
  - Humidity
  - Obstacle material properties
- Occupancy grids attempt to mitigate some of these

The Occupancy Grid

- Divide mapping area into a grid



The Occupancy Grid

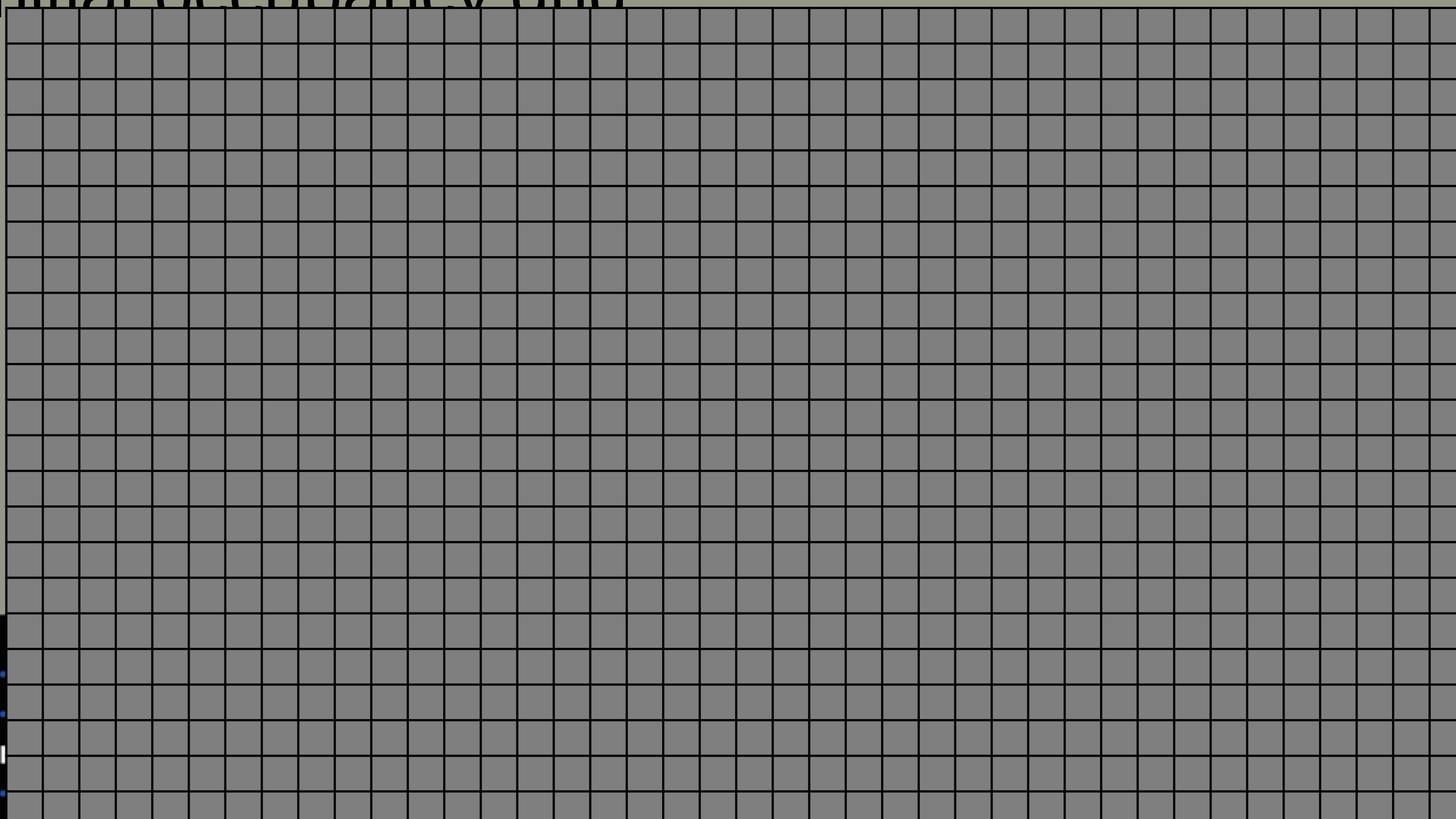
- Grid size is proportional to computational complexity:
  - How big is the area you are working in?
    - 5m X 5m
  - Let a grid square cover 5cm X 5cm area
    - 100 x 100 squares
    - 10000 squares

The Occupancy Grid

- So each grid square represents an area of some size
- Each square is assigned a probability
- This is the probability that the square is occupied
- 0 = definitely not occupied (white)
- 1 = definitely occupied (black)
- 0.5 = Equal chance of being occupied or unoccupied (grey)

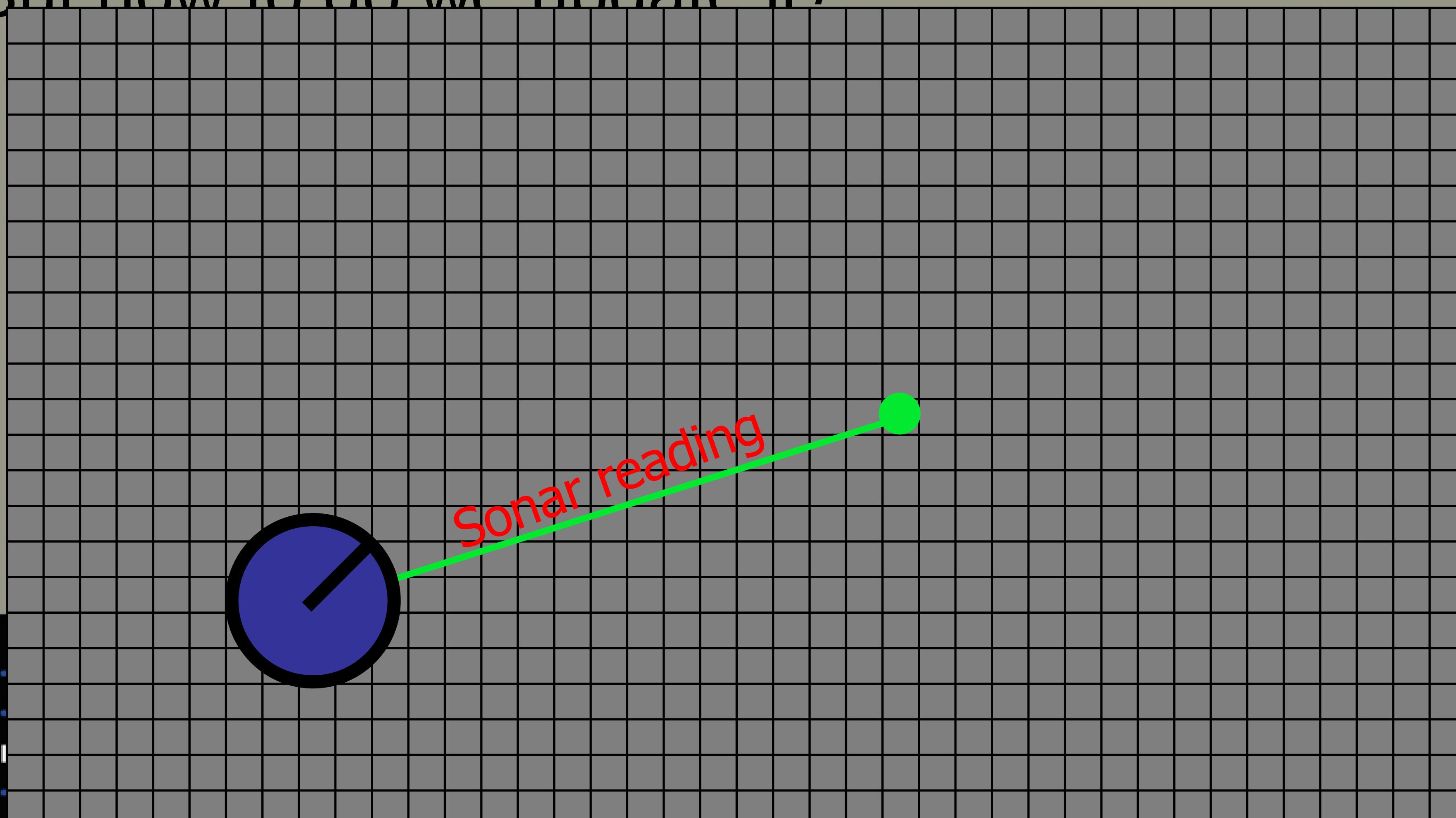
The Occupancy Grid

- Initial occupancy grid



The Occupancy Grid

- But how to do we update it?



The Occupancy Grid

- Apply our sonar model based on:
  - Current odometry position
  - Sonar readings
- Application uses Bayes' theorem

Bayes' Theorem

- **Conditional probability:**
- H is a hypothesis (something we wish to test the truth of), E is the available evidence, then:

$$p(H \| E) = \frac{p(E \| H) \times p(H)}{p(E)}$$

- p(E | H) is the likelihood of the data, given the hypothesis
- p(H) is the prior probability of the hypothesis.
- p(E) is the prior probability of the evidence (used to normalise the probabilities, 1-p(H))
- p(H | E) is the posterior probability of the hypothesis – the probability that H is true given the evidence E.

Bayes' Theorem

- **How does this relate to mapping?**
- The hypothesis is that a given grid square is occupied
- The occupancy grid holds the probability that a grid square is occupied – p(H) – prior probability
- p(E) is the second prior probability, the probability that the grid square is empty given by 1.0 – p(H)
- p(E | H) is the likelihood of the data, given the hypothesis given by the sonar model.
- p(H | E) is the posterior probability of the hypothesis – the probability that H (there is an obstacle) is true given the evidence E (new sonar reading).

Bayes' Theorem

- So we can now update the occupancy grid when we get a single reading
- If we only use this method we overwrite our hard gained evidence
- We need to use this previous evidence and **update** rather than **overwrite**

Recursive Bayes' Theorem

- Using the recursive form of Bayes' we get:

$$p(H \| E_t) = \frac{p(E_t \| H) \times p(H \| E_{t-1})}{p(E_t \| H) \times p(H \| E_{t-1}) + p(E_t \| \neg H) \times p(\neg H \| E_{t-1})}$$

- Terms are the same. t and t-1 refers to current time and previous time

Simple Worked Example

- Consider a grid square centred over 250,140
- Initial value is 0.5

- Sonar reading taken
- Sonar model gives this square p = 0.67

$$p(H|E_1) = \frac{0.67 \times 0.5}{0.67 \times 0.5 + 0.33 \times 0.5}$$

- New value = 0.67

Simple Worked Example

- Grid square centres over 250,140
- Value is 0.67

- Sonar reading taken
- Sonar model gives this square p = 0.71

$$p(H \| E_2) = \frac{0.71 \times 0.67}{0.71 \times 0.67 + 0.29 \times 0.33}$$

- New value = 0.83

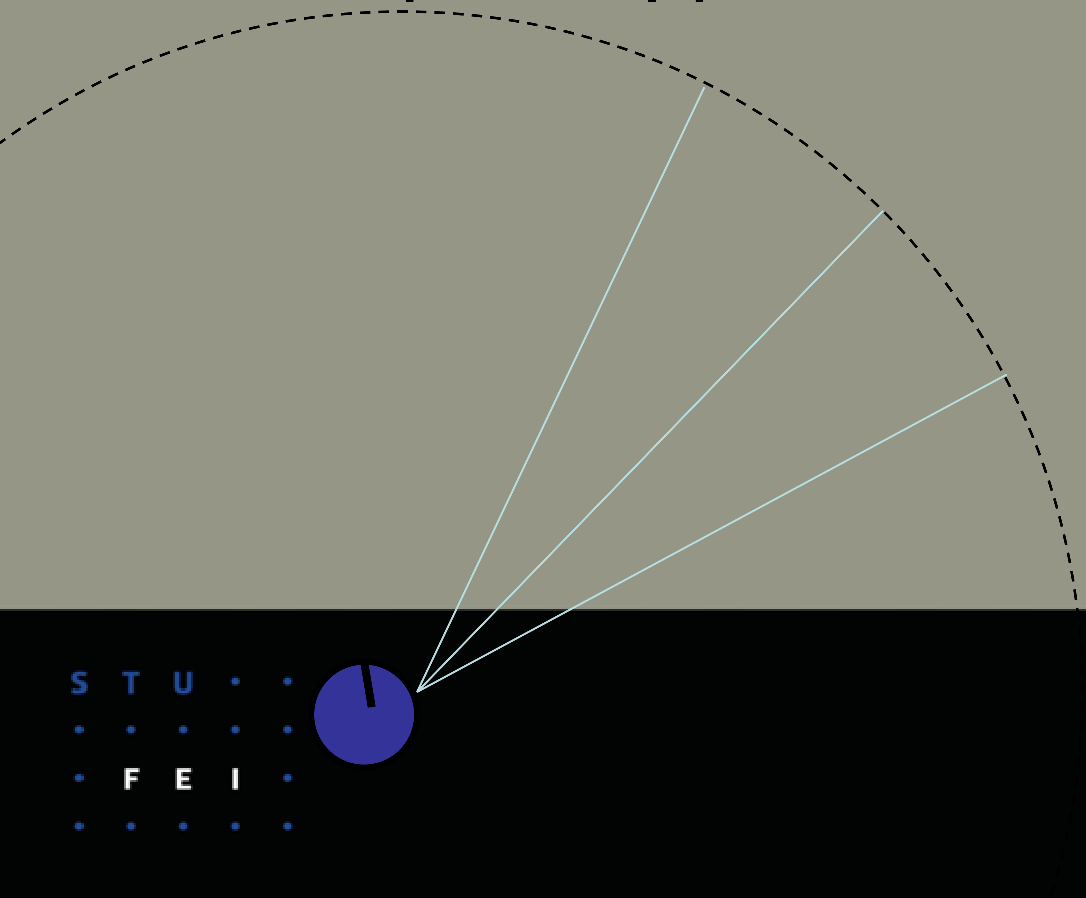
S T U
F E I

Simple Worked Example

- This calculation needs to be performed for every square inside the sonar ‘cone’

Implementation Details

- Sonar ‘cone’ bounding box
- One quick approach:



Implementation Details

- Sonar ‘cone’ bounding box
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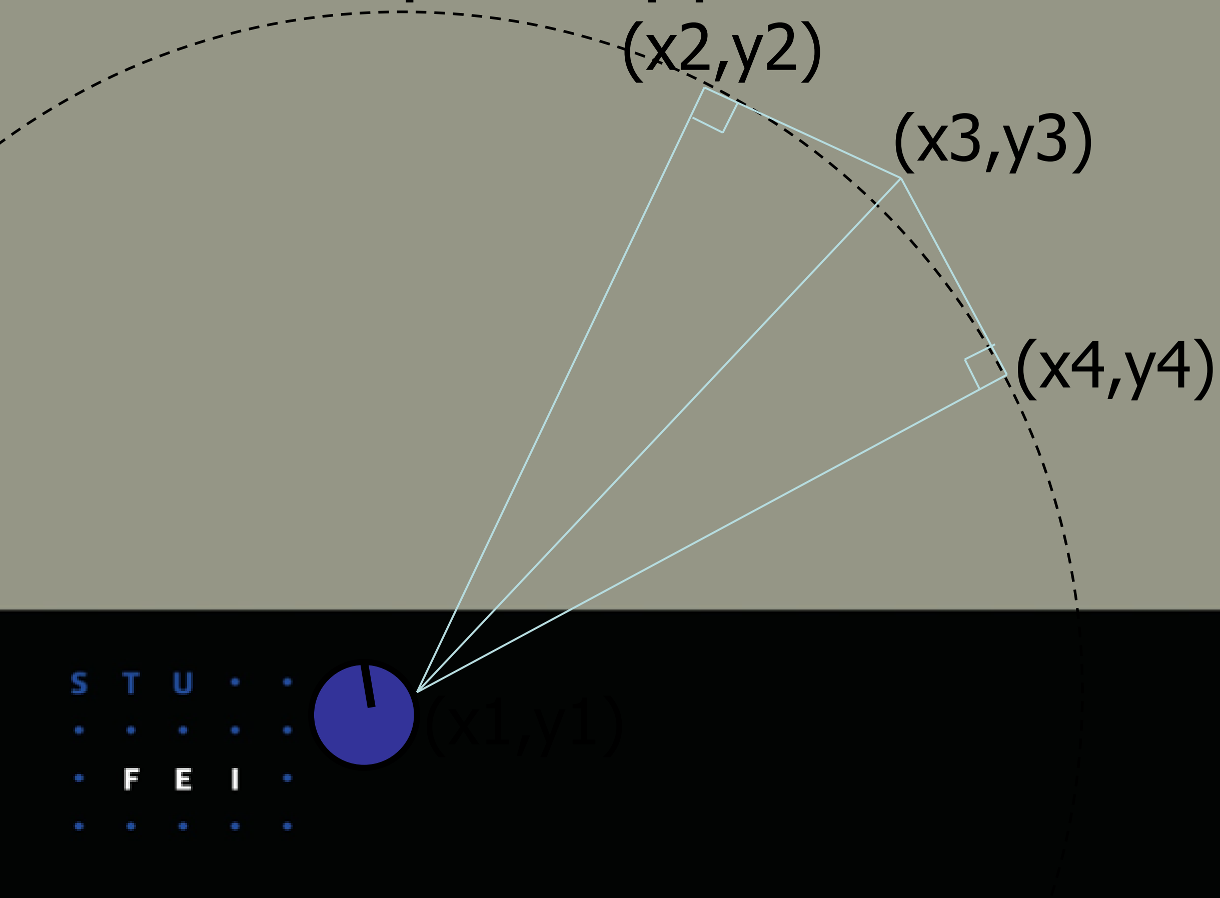

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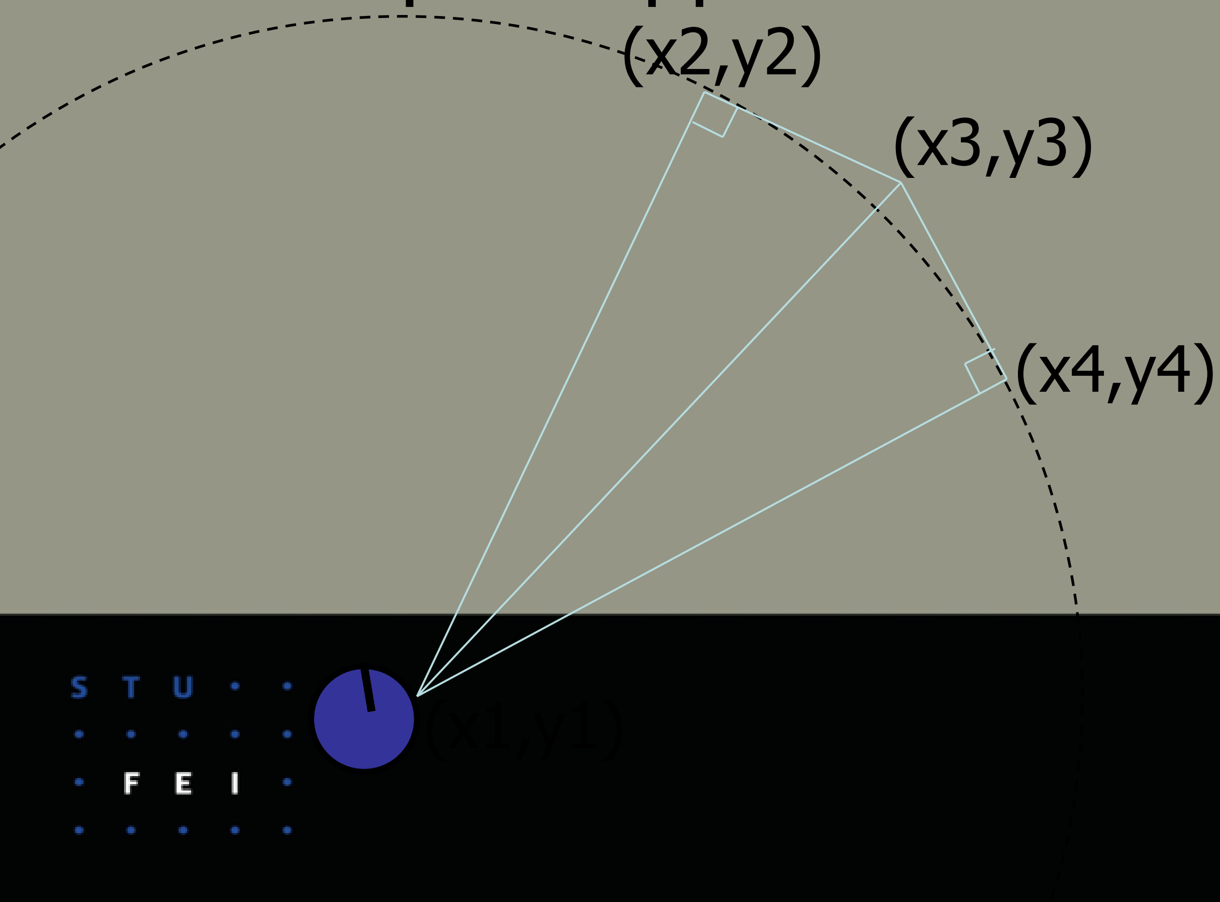
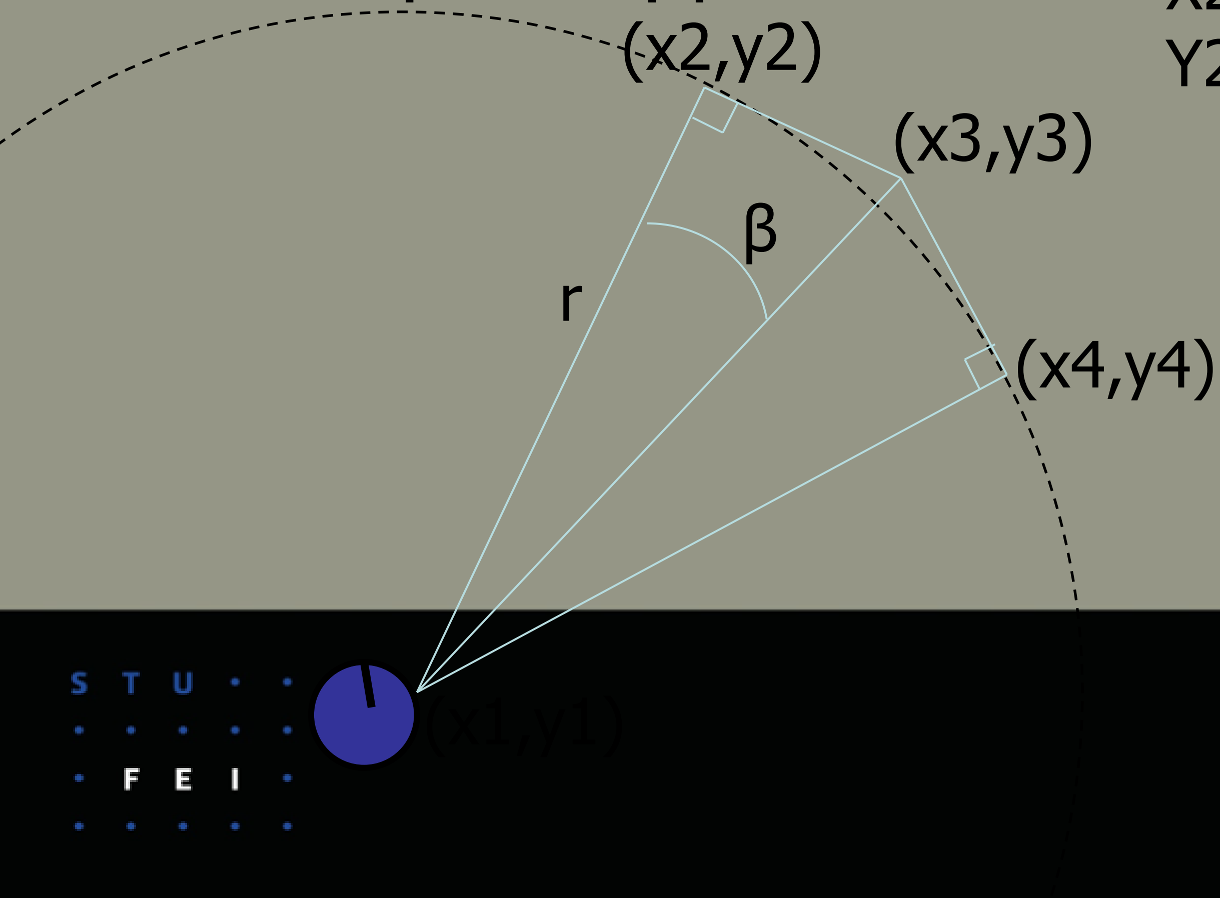


X1 = robotX + sonar offset
Y1 = roboyY + sonar offset

Implementation Details

- Sonar ‘cone’ bounding box
- One quick approach:

$$X2 = r \times \cos(sonarTh - \beta) + x1$$
$$Y2 = r \times \sin(sonarTh - \beta) + y1$$

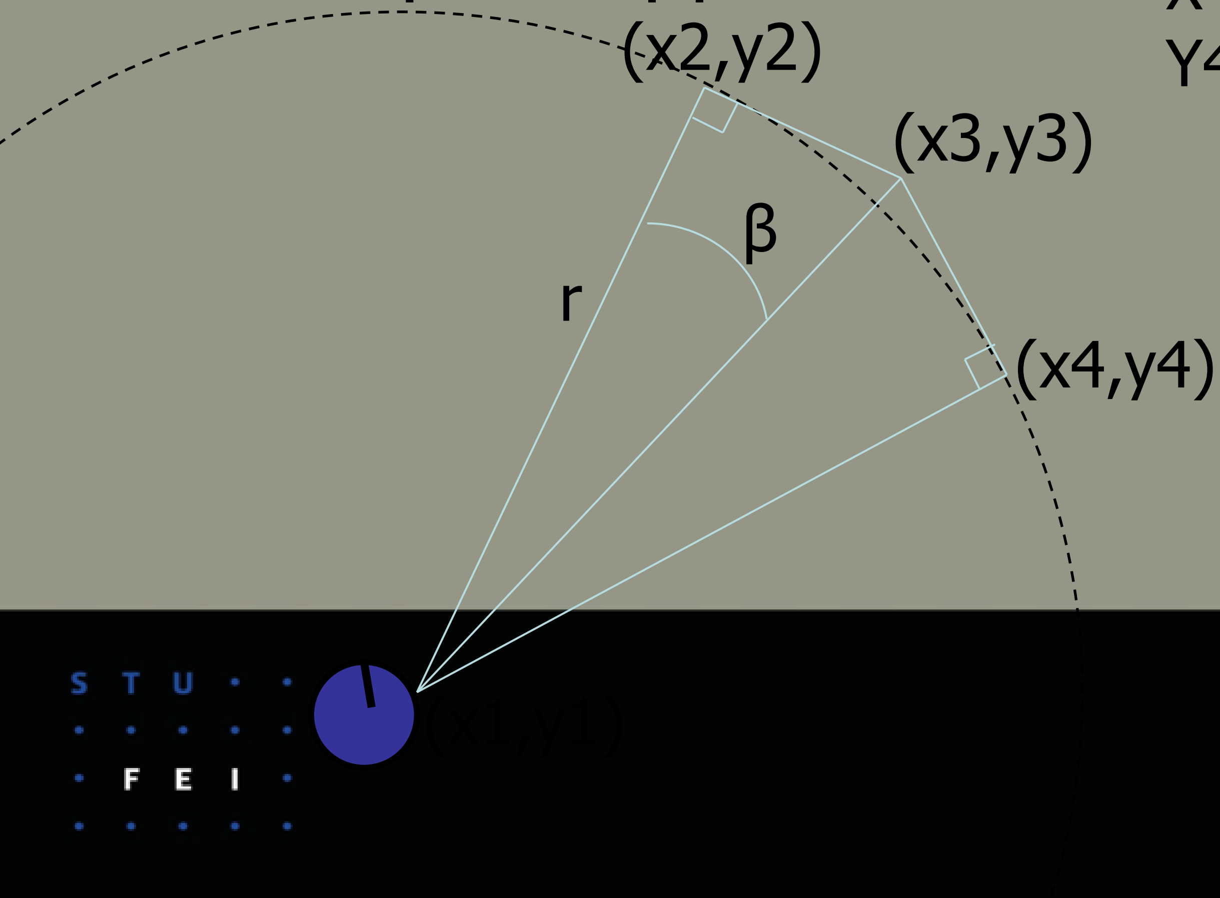


Implementation Details

- Sonar 'cone' bounding box
- One quick approach:

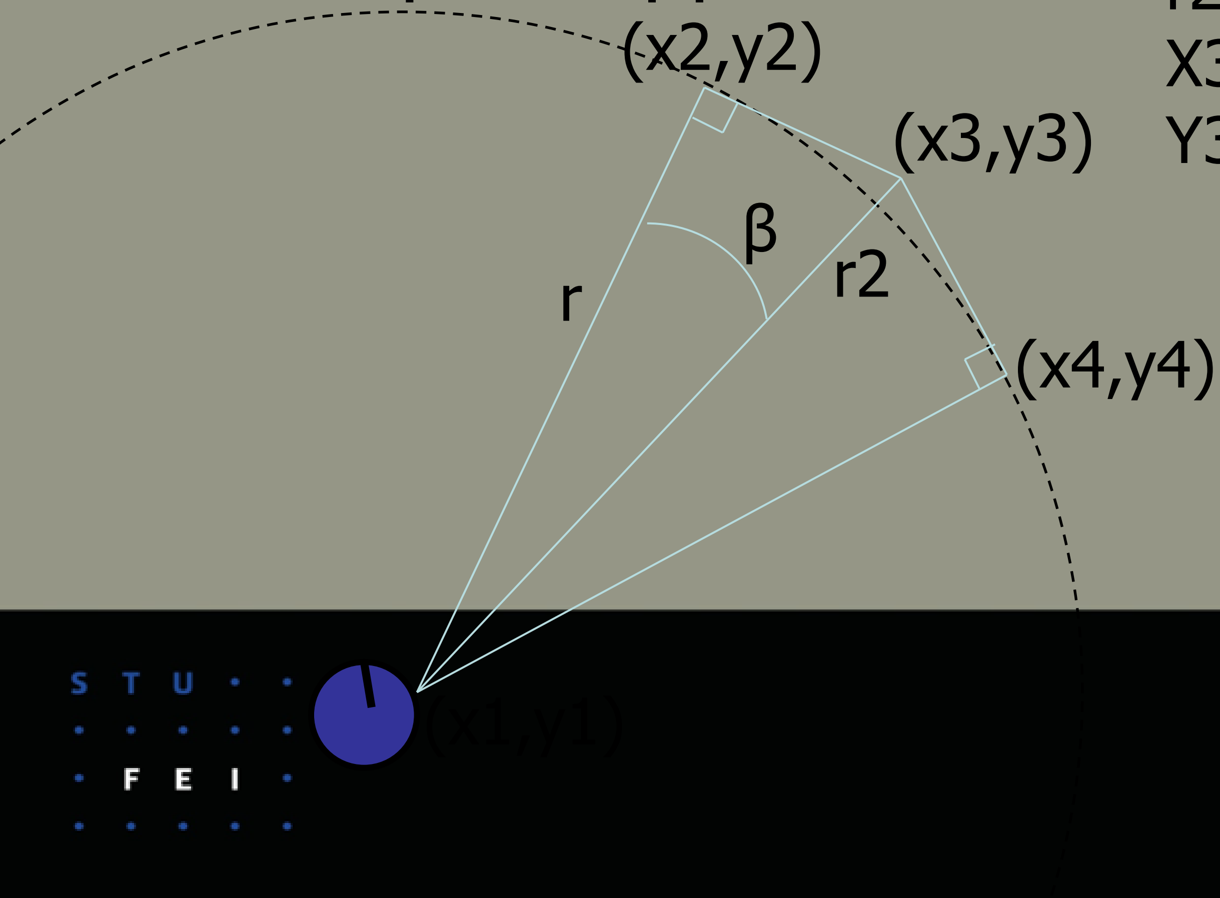
$X4 = r \times \cos(sonarTh + \beta) + x1$

$Y4 = r \times \sin(sonarTh + \beta) + y1$



Implementation Details

- Sonar 'cone' bounding box
- One quick approach:



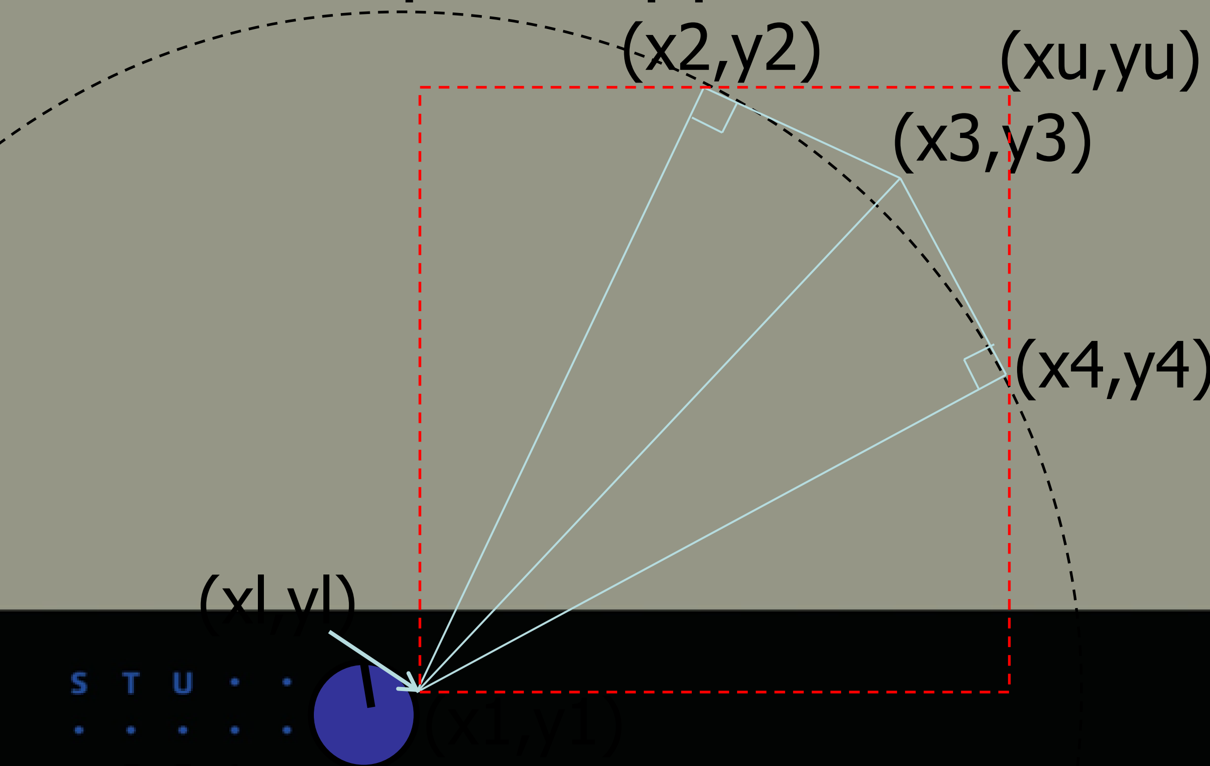
$r2 = r / \cos(\beta)$

$X3 = r2 \times \cos(sonarTh) + x1$

$Y3 = r2 \times \sin(sonarTh) + y1$

Implementation Details

- Sonar ‘cone’ bounding box
- One quick approach:



xl = min(x1,x2,x3,x4)
yl = min(y1,y2,y3,y4)

xu = max(x1,x2,x3,x4)
yu = max(y1,y2,y3,y4)

Which gives our BB

Can now write a for loop to iterate over the required squares in the grid

Implementation Details

- Sonar ‘cone’ bounding box
- Only iterate over grid squares in the BB
- Only update grid square values when they fall inside the sonar ‘cone’

Implementation Details

- Decision to be made
- We have multiple sensors
- Each with it's own pose
- Each has to be moved to a global pose
- Many options here
  - Option 1
  - Get sonar reading
  - Apply to grid
  - Rotate & translate grid
  - Repeat for all sensors

Implementation Details

- Decision to be made
- We have multiple sensors
- Each with it's own pose
- Each has to be moved to a global pose
- Many options here
  - Option 2
  - Get sonar reading
  - Rotate & translate
  - Apply to global grid
  - Repeat for all sensors

Implementation Details

- Decision to be made
- We have multiple sensors
- Each with it’s own pose
- Each has to be moved to a global pose
- Many options here
  - Option 3
  - Get sonar reading
  - Apply to single local grid
  - Repeat for all sensors
  - Rotate & translate
  - Apply to global grid

Occupancy Grids

- Demo

Applications

- An occupancy grid holds a probabilistic model of the environment built up from a number a sonar readings over time

- We can use this to identify our position using a process called Monte-Carlo Localization

Summary

- Robot perceptions are full of inaccuracies
- Application of probability techniques can mitigate the affect of these on decision making